SINGLE PULSE RESPONSE OF A PLASMA

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CALIFORNIA INSTITUTE OF TECHNOLOGY

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ABSTRACT

The conjecture by Hirschfield and Wachtel[1] that a weak pulse of radiation can trigger the emission of a large delayed burst of coherent electron cyclotron radiation is examined critically. The effect is found to exist when the electrons have a single velocity perpendicular to the magnetic field, but disappears in a thermal equilibrium distribution.

In attempting to understand the cyclotron resonance echoes observed by Hill and Kaplan[2], Hirschfield and Wachtel have suggested a very interesting single pulse mechanism which results in large coherent delayed bursts of electron cyclotron radiation. The mechanism requires an extremely homogeneous magnetic field and an energy-dependent cyclotron frequency, such as that provided by the relativistic mass change. For electrons with a single velocity perpendicular to the field, v_o, the current developed in plasma at the time of the <u>first</u> (of the many) delayed response was $|J| = .58 \ Nev_o$ where N is the number of electrons. This current is <u>independent of the electric field</u> required to trigger the burst, but the time at which the burst occurs is <u>inversely</u> proportional to the trigger field amplitude[1]. Hirschfield and Wachtel assert that a similar effect occurs when the electrons have a Maxwellian velocity distribution except that only the <u>first</u> of the multiple bursts survives. In this note we rederive their result for mono-energetic electrons, then show that, upon integration over a Maxwellian velocity distribution, the effect disappears. Only a monotonically decaying response to the trigger pulse, which is proportional to the trigger pulse amplitude, survives.

It is convenient to describe the time-development of the electron velocities in a rotating velocity space which rotates with angular velocity ω of the applied electric field[3,4]. The electric field is simply constant and produces a translation $v_1 = \frac{eE}{2m} t_{pulse}$ of each electron. We may neglect relativistic effects during the pulse provided that $v_1^2 << c^2$ and that the relativistic change in phase during the pulse $(\Delta\omega_c)_{rel.} t_{pulse}$, is negligible. Relativistic phase changes following the pulse can be substantial even though the relativistic change in cyclotron frequency is small, simply because the time involved may be large. Prior to the pulse $\check{v} = v_{\perp} e^{i\phi_o}$, where $v_{\perp}$ and ϕ_o are the electron's initial speed and phase, respectively. The real and imaginary parts of $\check{v}$ give v_x and v_y, respectively, in the rotating velocity space. Immediately following the pulse $\check{v} = v_{\perp} e^{i\phi_o} + v_1$. In the time interval t following the pulse the electron rotates through an angle

$$\phi \cong \omega_{co} t \left(1 - \frac{v^2}{2c^2}\right) - \omega t \tag{1}$$

where $v^2 = v_{\perp}^2 + 2v_o v_1 \cos \phi_o + v_1^2 + v_{\parallel}^2$.

The square of the electron speed is assumed small compared to c^2 and only the leading term in the relativistic mass change is employed. The (complex) current at time t is given by

$$\check{J} = -e \int dn[v_{\perp} e^{i\phi_o} + v_1]\, e^{i\phi} \tag{2}$$

where the integral is over the distribution of electrons.

We consider two cases: a) monoenergetic electrons, random phases, $dn = (N/2\pi)\delta(v_{\perp} - v_o)\ \delta(v_{\parallel})\ dv_{\perp} dv_{\parallel} d\phi_o$, and b) Maxwell distribution of electron velocities, $dn = N(\pi v_{th}^2)^{-3/2} e^{-(v_{\perp}^2 + v_{\parallel}^2)/v_{th}^2}\ v\ dv_{\perp} dv_{\parallel} d\phi_o$.

For monoenergetic electrons (2) becomes

$$\check{J} = - Ne\ e^{i\omega_c' t} \int_0^{2\pi} \left[v_o e^{i\phi_o} + v_1\right] e^{-i\omega_{co} t \frac{v_o v_1}{c^2} \cos \phi_o} \frac{d\phi_o}{2\pi}$$

where $\omega_c' = \omega_{co} \left[1 - \dfrac{v_o^2 + v_1^2}{2c^2}\right] - \omega$.

Using the identity $e^{-i\alpha \cos \phi_o} = \sum_{-\infty}^{\infty} (-i)^n J_n(\alpha)\, e^{-in\phi_o}$ and performing the ϕ_o integration yields

$$\check{J} = - Ne\, e^{i\omega_c' t} \left[-iv_o J_1\left(\omega_{co} t \frac{v_o v_1}{c^2}\right) + v_1 J_o\left(\omega_{co} t \frac{v_o v_1}{c^2}\right) \right] \qquad (3)$$

This is the result of Hirschfield and Wachtel[1]. When $v_1 << v_o$ the first term is generally more important [except at times when $J_1(\omega_{co} t \frac{v_1 v_1}{c^2})$ is zero] , the main effect of the small second term being to produce a small change in the zero-crossing times of the quantity in brackets. The important result is that the magnitude of the current exhibits periodic maxima, the first occurring when $\omega_{co} t \frac{v_o v_1}{c^2} = 1.84$ with a current $|J| = .58\ Ne\ v_o$. The maximum current corresponds to a very strong phase bunching and is <u>independent of</u> v_1 , although the time required for the maximum current to develop increases as v_1 is decreased. This result indicates that a very weak pulse could trigger the emission of a very <u>intense</u> burst of cyclotron radiation in which most of the particles radiate coherently.

Hirschfield and Wachtel[1] have indicated that the main change introduced by assuming a Maxwellian distribution of electron velocities is to eliminate the secondary maxima, but that the first maximum remains and its characteristics may be estimated by using v_{th} , the electron thermal speed, for v_o in the above formulas. While at first sight this modification has an element of plausibility, the end result, that a weak trigger pulse can cause a nearly complete phase bunching of an equilibrium plasma, appears to violate the second law of thermodynamics. We now show that this result is not correct.

When we assume dn to be given by a Maxwellian distribution [assumption (b)] we obtain

$$\check{J} = - \mathrm{N}ev_{th}\, I_{\|}(t) \int \frac{2v_\perp dv_\perp}{v_{th}^2}\, e^{v_\perp^2 \left(\frac{1}{v_{th}^2} + \frac{i\omega_{co} t}{2c^2}\right)} \left[-i \frac{v_\perp}{v_{th}} J_1\!\left(\omega_{co} t \frac{v_\perp v_1}{c^2}\right) + \right.$$

$$\left. + \frac{v_1}{v_{th}} J_o\!\left(\omega_{co} t \frac{v_\perp v_1}{c^2}\right)\right] \tag{4}$$

where
$$I_{\|}(t) = \frac{1}{\sqrt{\pi}} \int_{-o}^{\infty} e^{-v_{\|}^2 \left(\frac{1}{v_{th}^2} + \frac{i\omega_{co} t}{2c^2}\right)} \frac{dv_{\|}}{v_{th}} = \frac{1}{\sqrt{1 + i\frac{\omega_{co} t\, v_{th}^2}{2c^2}}} \tag{5}$$

comes from the integration over the component of velocity parallel to the magnetic field. The integral over perpendicular speeds can be accomplished with the integral

$$\left[\int_0^\infty J_\nu(ax)\, e^{-p^2x^2}\, x^{\nu+1}\, dx = a^\nu e^{a^2/4p^2} / (2p^2)^{\nu+1} \right]$$

and yields:

$$\check{J}/(-Ne\, v_{th}) = \left[\frac{1}{\sqrt{1+i\tau}}\right]\left[\frac{V_1}{1+i\tau} - \frac{i\tau V_1}{(1+i\tau)^2}\right] e^{-V_1^2\, \tau^2/(1+i\tau)} \tag{6}$$

where the current is expressed in units of $Ne\, v_{th}$, the "thermal current", $\tau = \omega_{co} t\, v_{th}^2/c^2$ is the time expressed in units of the coherence time of cyclotron orbits, and $V_1 = v_1/v_{th}$ is the strength of the "trigger" pulse expressed in units of the thermal speed. The first factor arises from the integration over parallel velocities and contributes a decay $\sim \tau^{-1/2}$ when $\tau \gg 1$. The first and second terms in the second bracket arise from the integration of the Bessel functions J_o and J_1 in (3) respectively. The important thing to note is that both contributions are now proportional to $\underset{-}{V}_1$ and that they combine in a simple way. Thus the plasma current induced by the pulse is strictly proportional to the amplitude of the pulse and furthermore the magnitude of the current decays monotonically from the initial value caused by the pulse:

$$\left| \frac{J}{Ne\ v_{th}} \right| = \frac{V_1}{(1+\tau^2)^{5/4}}\, e^{-V_1^2 \tau^2/(1+\tau^2)} \qquad (7)$$

When $V_1^2 \ll 1$ (weak pulse), the decay is governed by the first algebraic factor and the decay time is therefore

$$t_{decay} \approx \frac{1}{\omega_{co}} \frac{2c^2}{v_{th}^2}$$

The coherence time for cyclotron orbits (dephasing) arises because the cyclotron frequency depends slightly on particle energy and there is a distribution of energies.

One could therefore say that Hirschfield and Wachtel's mechanism fails to exist in a thermal plasma because the time required for phase bunching by their process exceeds the time for thermal dephasing by a factor $v_{th}/v_1 \gg 1$, and the latter process dominates. As a qualitative extension of this notion, one can also say that for the effect to exist in an almost monoenergetic distribution of perpendicular energies, such as in the experiment by Wachtel and Hirschfield[5], the bunching time must be less than the thermal dephasing time corresponding to the spread in velocities. This in turn sets a lower limit on the pulse strength, i.e., $v_1 > \Delta v_\perp$, where $\Delta v_\perp$ is a measure of the spread in perpendicular velocities.

In a similar fashion we can obtain a criterion for the required homogeneity of the magnetic field. When there is a spread in cyclotron frequencies of particles in the region of observation, this gives rise to an additional dephasing. In order that the one pulse maximum not be eliminated by magnetic field inhomogeneities $\Delta\omega_c \ll 1/t_{max}$, or

$$\Delta\omega_{co} \ll \omega_{co} \frac{v_o v_1}{c^2} \tag{8}$$

where $\Delta\omega_{co}$ denotes the spread in cyclotron frequencies due to magnetic field inhomogeneities. This expression is also seen to imply a lower limit for v_1, for a given $\Delta\omega_{co}$.

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REFERENCES

1. J. L. Hirschfield and J. M. Wachtel, 7th Ann. Am. Phys. Soc. Meeting Div. Plasma Physics 8-11 November 1965, Paper A-2, Bull. Am. Phys. Soc. (to be published).

2. R. M. Hill and D. E. Kaplan, Phys. Rev. Letters 14, 1062 (1965).

3. Roy W. Gould, Phys. Letters 19, 477 (1965).

4. Roy W. Gould, Bull. Am. Phys. Soc. 10, 1191 (1965); and Technical Report No. 28, Nonr 220(50), Dec. 1965.

5. J. M. Wachtel and J. L. Hirschfield, Bull. Am. Phys. Soc. 11, 375 (1966).

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